

Year 12 Mathematics Specialist 2019

Test Number 5:

Rates of Change and Differential Equations

Resource Free

Name: _____

Teacher: DDA

Marks: 28

Time Allowed: 25 minutes

Instructions: No notes or calculators. Show your working where appropriate remembering you must show working for questions worth more than 2 marks.

Question 1**[2 marks]**

What is $\frac{dy}{dx}$ for $3y^2 - 5x^3 = 4x$?

Question 2**[2 marks]**

Given $y = 4x^2 + 5x$ and $\frac{dx}{dt} = 2$ when $x = 1$, find $\frac{dy}{dt}$ when $x = 1$.

Question 3**[1 marks]**

What is the general solution to $\frac{dy}{dt} = 30(2t + 1)^2$?

Question 4**[4 marks]**

A train leaving the station has its acceleration given by $1.8t - 0.21t^2$ for the first 10 seconds of motion. What is the speed and distance moved after 10 seconds?

Question 5**[4 marks]**

Solve $\frac{dy}{dx} = (2x - 5)(y + 3)$.

Question 6**[3 marks]**

Consider the differential equation $\frac{dy}{dx} = x + y - 2$. Let $y = g(x)$ be the solution to the differential equation with initial condition $g(-1) = k$ where k is a constant. Euler's method, starting at $x = -1$ with step size 1, gives the approximation $g(2) \approx -1$. Find the value of k .

Question 7**[5 marks]**

Find the equation of the tangent to the curve $e^y + y \ln x = 3$ at the point $P(1, \ln 3)$.

Question 8**[3 marks]**

The three slope fields shown below have their associated differential equations in the following list. Choose the correct equation for each slope field.

$$\frac{dy}{dx} = 2y$$

$$\frac{dy}{dx} = y$$

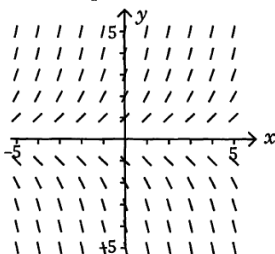
$$\frac{dy}{dx} = x$$

$$\frac{dy}{dx} = (x-2)(y-3)$$

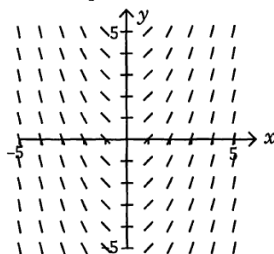
$$\frac{dy}{dx} = \frac{x-3}{y-2}$$

$$\frac{dy}{dx} = (x-3)(y-2)$$

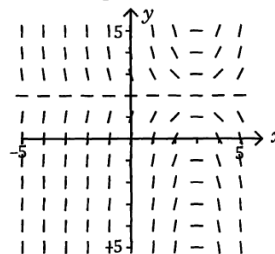
Slope field A.



Slope field B.



Slope field C.

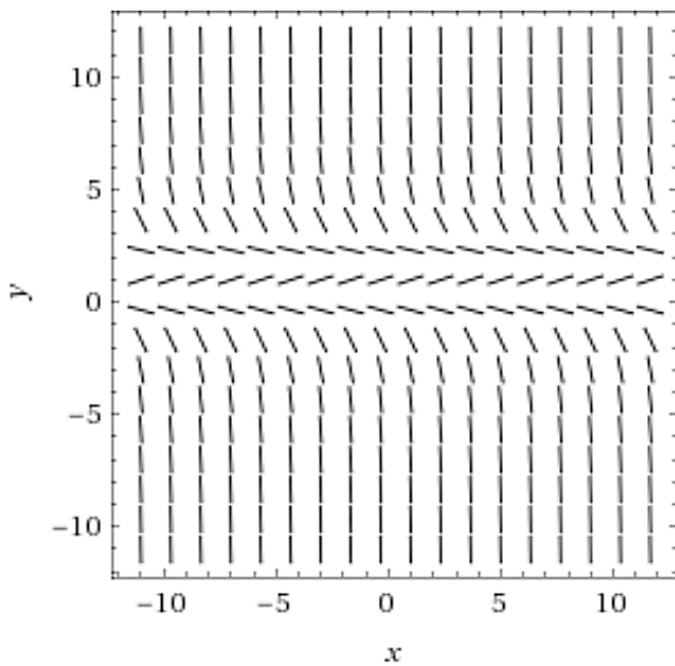


Question 9

[2 marks]

Would the differential for the slope field below be of the form $\frac{dy}{dx} = f(x)$ or $\frac{dy}{dx} = f(y)$?

Give a reason for your answer.

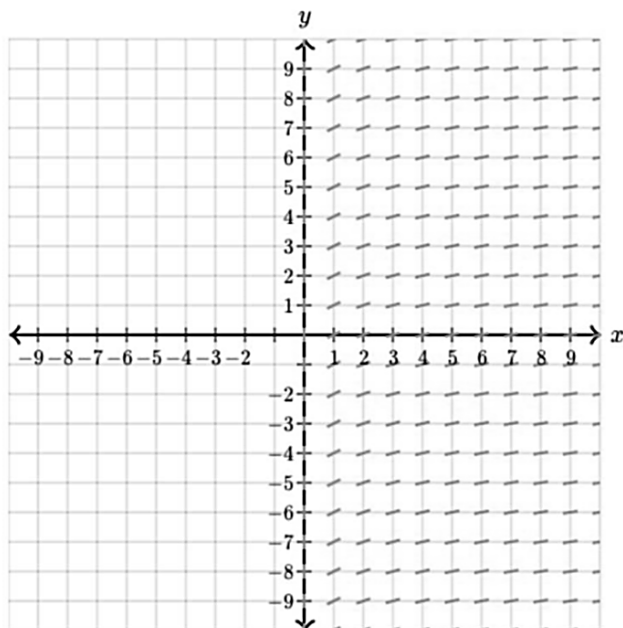


Question 10

[2 marks]

For the slope field below, draw in the particular solution given that the initial condition (1,7).

What form would the solution to this equation be in?

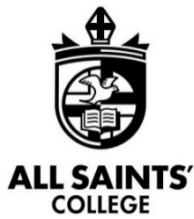


Year 12 Mathematics Specialist 2019

Test Number 5:

Rates of Change and Differential Equations

Resource Rich



Name: _____

Teacher: DDA

Marks: 21

Time Allowed: 20 minutes

Instructions: You are permitted 1 A4 page of notes and your calculator. Show your working where appropriate remembering you must show working for questions worth more than 2 marks.

Question 11**[4 marks]**

A particle with SHM is moving with a velocity of 3.6 m/s as it passes through its central position. When the particle is 0.2 m from the central position, it has an acceleration of 4.8 m/s^2 . Calculate the amplitude and period of the oscillation.

Question 13**[6 marks]**

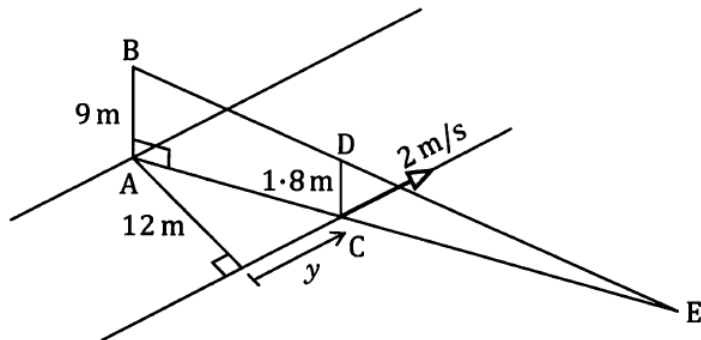
Ben has a young son who was recently ill with a fever. He noticed that after being given a dose of penicillin the child's temperature increased, peaked and then decreased. Ben approximated the child's temperature above 37° by the function $T(x) = xe^{-0.2x}$ where $x > 0$, where x refers to the time in hours after 7.00pm. Using this model find:

- a) The rate of change of temperature and show that for $0 < x < 5$, the temperature was increasing.
- b) The maximum temperature and the time this occurred.

Question 14

[6 marks]

The diagram, not shown to scale, shows a lamp post, AB, of height 9 metres, situated on one side of a straight road.



The road is of width 12 metres and line CD, see diagram, represents a person of height 1.8 metres walking at 2 metres/second along the other side of the road from the lamp. The light at B causes the person to have a shadow shown as CE. Find the rate at which the length of the shadow is changing at the instant when AC is of length 20 metres.